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Technical Notes

Effect of the Combined Burning-Surface/ Particle Motion on Aluminum Agglomeration

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Nomenclature

a	=	burning-rate constant
k	=	constant [Eq. (9)]
k_p	=	volume coefficient of infill
L	=	burning-surface regressed distance
m	=	constant [Eq. (9)]
n	=	burning-rate exponent
$n_{\max}$	=	maximum number of particles per unit area of the burning surface
n_V	=	average number of particles per unit volume
n_0	=	average number of particles per unit area of the burning surface
R	=	particle radius
T_{BS}	=	burning-surface temperature
T_0	=	interior propellant temperature
t	=	time
V	=	moving boundary-surface velocity
v	=	volume fraction of particles in the solid propellant
λ	=	condensed-layer thickness

Subscripts

c	=	coalescence
f	=	complete filling
max	=	maximum value

I. Introduction

ALUMINUM powders (typically of 5–20 μm radius) are added to solid propellants to obtain higher burning rates and increase the energetic performance. In these propellants, assuming ideal burning implies that all particles stay immobile in the propellant volume before meeting the regressing burning surface (BS) and they are ejected (lifted) from the burning surface into the combustion chamber without interacting with the burning surface. However, the motion of the 2-D moving burning surface and its interaction with the mobile particles can lead to diffusion motion of the particles. The burning surface moves together with the aluminum particles, and this process results in particle accumulation and agglomeration on or near the propellant burning surface [1–9], as shown in Fig. 1.

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The problem of aluminum agglomeration has been the subject of numerous experimental and theoretical studies, yet there is still disagreement regarding the mechanism that controls particle agglomeration. Two major models can be considered as representative: the packing model (Jackson et al. [9]) and the pocket model (Beckstead [10] and Cohen [11]).

Below the burning surface of the solid propellant, a condensed-phase regressing layer of thickness λ exists, moving at velocity V, as shown in Fig. 1. The condensed layer can be a real layer of molten binder with particles packed [1,9] or even a pocket [10,11]. The temperature in the propellant decreases from $T = T_{BS}$ on the burning surface to $T = T_0$ at the lower end of the condensed layer [1–9].

The motion of the 2-D boundary–particle complex in any matrix containing stable particles leads to collisions and coalescing of the particles inside the matrix. According to the theoretical model of Geguzin et al. [12,13], there are two regimes of combined motion: particle sweeping and particle coalescence.

In the present work, a simplified theoretical model for the burning-surface–particle motion is proposed. The model is based on the geometrical parameters of the system that contains the burning surface and the particles; it takes into account the specific physical parameters of the system, and it is consistent with experimental data on particle diffusion motion. The possible regimes of the combined motion of the burning surface and the particles and the physical parameters influencing on the regimes are considered.

The aim of the present study is to provide physical explanations for experimentally observed phenomena using basic mathematical tools. The approach is mostly heuristic and the results of the model are qualitative.

II. Theoretical Model

It is assumed that the volume distribution of the particles is uniform and the moving 2-D burning surface (a plane) is arbitrarily oriented in space. The existence of ammonium perchlorate in the case of composite propellants is not taken into account. The average number of particles per unit area of the burning surface, n_0, the average number of particles per unit volume, n_V, and the maximal possible number of particles per unit area of the burning surface, $n_{\max}$, can be expressed by the equations [12–16]

$$n_0 = 3v/(2\pi R^2) \tag{1}$$

$$n_V = 3v/(4\pi R^3) \tag{2}$$

$$n_{\max} = 1/(2\sqrt{3}R^2) \tag{3}$$

At $t = 0$, the burning surface starts moving (together with the particles on it) along the plane perpendicular to the burning surface, passes some distance L, and sweeps particles inside the matrix, without particle collisions. Before $t = t_c$, the particles only accumulate on the boundary surface. The maximal possible values $L = L_{\max}$ and $t_c = t_{c,\max}$ correspond to a state in which the number of particles reaches its maximum value, $n_{\max}$; therefore, the following equations can be written:

$$n_{\max} = n_0 + n_V \cdot L_{\max} \tag{4}$$

$$1/(2\sqrt{3}R^2) = 3v/(2\pi R^2) + 3vL_{\max}/(4\pi R^3) \tag{5}$$

Assuming $V = \text{const}$ at constant-combustion-chamber pressure p conditions, Eqs. (1–5) give

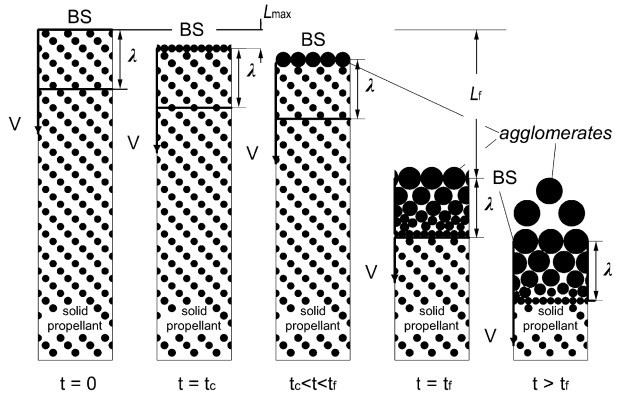

Fig. 1 Schematic of agglomerate formation in a particle-laden solid propellant during burning.

$$L_{\max} = \frac{2R(\pi\sqrt{3} - 9v)}{9v} \tag{6}$$

$$t_{c\max} = \frac{L_{\max}}{V} = \frac{2R(\pi\sqrt{3} - 9v)}{9v \cdot V} \tag{7}$$

In the present case, a typical regression rate of $V \approx 5 \times 10^{-3}$ m/s [1,5,8,9] can be used.

A summary of data calculated from Eqs. (1–7) for solid-propellant burning is given in Table 1.

III. Discussion

Let us consider the case of the burning-surface–particle complex motion after the time of collision commencement ($t > 2.2 \times 10^{-2}$ s), assuming a particle size smaller than the condensed-layer thickness ($2R < \lambda$) (Fig. 1). During its movement, the burning surface sweeps particles inside the condensed layer, including those that penetrated into the layer from the solid boundary. The process of particle sweeping inside the layer consequently leads to particle accumulation, collisions, and formation of agglomerates. Note that the temperature increase leads to a decrease in the aluminum particle coalescence time [6], and since there is a temperature gradient in the solid propellant, it can be assumed that the kinetics of particle coalescence are more intense on the burning surface. Therefore, the formation of agglomerates on the burning surface that are larger than those at the bottom of the condensed layer can be expected.

In the condensed layer, both particles and agglomerates have high mobility (particle mobility in liquids), and it can be assumed that the particle sweeping proceeds without any critical change until $t = t_f$ (complete filling of the layer by particles). After that ($t > t_f$), the particles inside the solid propellant cannot easily penetrate into the condensed layer; hence, the particles at the bottom boundary of the condensed layer pose a barrier to the combined motion of the mobile layer, the particles, and the agglomerates. A consequence can be the lifting of the agglomerates from the mobile layer toward the gaseous phase. Obviously, the earlier the lifting begins (i.e., small t_f values), the smaller the agglomerates. In the extreme case of particles with initial radius $2R > \lambda$, lifting of particles should be expected in the beginning of propellant burning, and so it can be deducted that agglomeration of large-size particles ($2R > \lambda$) is insignificant.

To estimate the maximum possible value of the filling time $t_f = t_{f,\max}$ for particles of radius $2R < \lambda$, it is assumed that complete filling by particles corresponds to the closest packing of balls in the condensed layer, which is determined by the distance passed by the burning surface: $L_f = V \cdot t_{f,\max}$. The volume coefficient of infill for closest packing of balls, k_p, is known from crystallography ($k_p \approx 0.7405$ for particles of any radius); therefore, using Eqs. (1–6), the following relation can be obtained:

$$k_p \cdot \lambda \approx 0.7405 \cdot \lambda \approx \tfrac{4}{3}\pi R^3 \cdot n_0 + v \cdot L_f \approx 2v \cdot R + v \cdot L_f \tag{8}$$

It can be assumed that the values of λ and V depend on the combustion-chamber pressure by the relations

$$\lambda = k \cdot p^{-m} \tag{9}$$

$$V = a \cdot p^n \tag{10}$$

where a, k, m, and n are positive constants [1,2,4,5]. Then the value of $t_{f,\max}$ can be obtained using Eqs. (8–10):

$$t_{f,\max} = \frac{L_f}{V} = \frac{0.7405\frac{\lambda}{v} - 2R}{V} = \frac{0.7405\frac{kp^{-m}}{v} - 2R}{ap^n} \tag{11}$$

The main characteristics of the agglomeration process (for particles of size $2R < \lambda$) can be identified, checking the sign of the partial derivatives of the filling time $t_{f,\max}$ in Eq. (11), with respect to parameters p, v, and R. Assuming a typical solid propellant with characteristic values $\lambda = 10$–100 μm, $v = 0.01$–0.1, and $R = 5$–20 μm and considering the fact that a, k, m, and n are positive, the partial derivatives $(\partial t_{f,\max}/\partial p)_{v,R}$, $(\partial t_{f,\max}/\partial v)_{p,R}$, and $(\partial t_{f,\max}/\partial R)_{v,p}$ are negative. Based on this fact and assuming that t_f is smaller than the $t_{f,\max}$ value obtained by Eq. (11), the following conclusions can be drawn regarding particle agglomeration:

1) Increasing the pressure leads to a decrease in the t_f value and, as a consequence, to a decrease in the agglomerate size (for constant particle radius and volume fraction). This has been confirmed experimentally in [1,5,8,9].

2) The agglomerate size is larger for the propellant containing the smaller volume fraction of particles (for constant particle radius and chamber pressure). This is consistent with the experimental findings of Gany and Caveny [1], which indicate that agglomeration of large-size particles requires very high volume fraction.

3) Decreasing the particle radius results in an increase in agglomerate size (for constant particle volume fraction and chamber pressure). This is also in agreement with the experimental results of Gany and Caveny [1], although this is still an open issue.

4) Gany and Caveny [1] indicate that for particles larger than the condensed-layer thickness ($2R > \lambda$), a sharp decrease in agglomeration occurs. Prominent agglomeration takes place for particle diameters smaller than the layer thickness ($2R < \lambda$). This agrees with the theoretical findings of the present study.

Table 1 General parameters of a planar burning surface moving together with the particles

Parameter	Value
$t_{c\max}$, s	2.2×10^{-2}
R, μm	5
v	0.05
n_0, m^{-2}	10^9
n_V, m^{-3}	10^{14}
$n_{\max}$, m^{-2}	2×10^{10}
$L_{\max}$, μm	111

IV. Conclusions

A qualitative model has been developed for aluminum particle agglomeration. Assuming a uniform distribution of the particles, the combined motion of the burning surface in solid propellants together with particles has been analyzed. The time needed for particle agglomeration depends on the velocity of the burning surface, the radius, and the volume fraction of the particles. It is assumed that the lifting of the agglomerates from the burning surface toward the gaseous phase occurs after complete filling of the condensed layer by particles. The obtained conclusions are in qualitative agreement with experimental data.

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